Dispersive approach: application to \( \omega/\phi \rightarrow 3\pi \)

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  \[ \omega/\phi \to 3\pi \] (see the talk of Peng Guo about \( \eta \to 3\pi \))
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Aim
**Motivation**

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Spectrum of QCD:
- complete understanding,
- discover new resonances,
- exotics ...
**Motivation**

Spectrum of QCD: complete understanding, discover new resonances, exotics ...

Lattice: Jozef J. Dudek
Phys.Rev. D84 (2011) 074023
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Decay properties of the
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Motivation

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Decay properties of the known states

- multi-body (final state) interactions are expected to play a crucial role for the hadron spectroscopy
- analysis of the precision exp. data
- test/develop/cross-check tools on conventional states → move to exotic
Decay properties of the known states

\( \omega \)-meson discovered in ~1960th

\( \pi p \rightarrow \omega p, Kp \rightarrow \omega \Lambda, e^+e^- \rightarrow 3\pi, pp \rightarrow \omega \pi\pi, \ldots \)

**number of events:** \(10^3 - 10^4\)
Decay properties of the known states

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$\pi p \rightarrow \omega p$, $Kp \rightarrow \omega \Lambda$, $e^+e^- \rightarrow 3\pi$, $pp \rightarrow \omega \pi\pi$, …

**number of events:** $10^3 - 10^4$

Dalitz plot: $\omega \rightarrow 3\pi$

$$
\frac{d^2 \Gamma}{ds \, dt} \propto |\vec{p}_+ \times \vec{p}_-|^2 |F(s, t)|^2
$$
Decay properties of the known states

ω-meson discovered in ~1960th

π p → ω p, Kp → ωΛ, e⁺e⁻ → 3π, pp → ωππ, ...

number of events: 10³ - 10⁴

Dalitz plot: ω→3π

\[
\frac{d^2\Gamma}{ds \; dt} \propto |\vec{p}_+ \times \vec{p}_-|^2 |F(s, t)|^2
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\[
x \propto (t - u)
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\[
y \propto (s_c - s)
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Not yet experimentally shown.
Decay properties of the known states

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\textbf{number of events: } 10^3 - 10^4

Dalitz plot: \( \omega \rightarrow 3\pi \)

\[
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\]

\[ x \propto (t - u) \]
\[ y \propto (s_c - s) \]

\[ x \]
\[ y \]

CLAS, WASA, KLOE
\textbf{number of events: } 10^6 - 10^7

not yet experimentally shown
Decay properties of the known states

$\eta \rightarrow 3\pi$

Slow convergence of ChPT: \[ \Gamma_{\eta \rightarrow \pi^+\pi^-\pi^0} = 66 \text{ [LO]} + 94 \text{ [NLO]} + \ldots = 296 \pm 16 \text{ eV [Exp]} \]
Decay properties of the known states

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Slow convergence of ChPT: $\Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} = 66 \text{ [LO]} + 94 \text{ [NLO]} + \ldots = 296 \pm 16 \text{ eV [Exp]}$

Slope parameter puzzle for $\eta \rightarrow 3\pi^0$

$$|A_{\eta \rightarrow 3\pi^0}|^2 \propto 1 + 2\alpha z + \ldots$$

Schneider et al.

JHEP, 1102:028, (2011)
Decay properties of the known states

\( \eta \to 3\pi \)

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Extraction of u-and d-quark mass difference?

\[
\mathcal{L}_{IB} = \frac{m_u - m_d}{2}(\bar{u}u - \bar{d}d)
\]

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\( \eta \rightarrow 3\pi \)

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New data from WASA at COSY (2014)

1.2\times10^7 decays

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New data from WASA at COSY (2014) $1.2 \times 10^7$ decays

Schneider et al. JHEP, 02:028, (2011)

Talk of Peng Guo
<table>
<thead>
<tr>
<th>Particle</th>
<th>$e^+ e^- \gamma$</th>
<th>$\pi^+ \pi^- \gamma$</th>
<th>$\pi^+ \pi^- \pi^0$, $\pi^+ \pi^- e^+ e^-$</th>
<th>$\pi^+ \pi^- \eta$, $\pi^+ \pi^- e^+ e^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
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<td></td>
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</tr>
<tr>
<td>$\eta$</td>
<td>$e^+ e^- \gamma$</td>
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</tr>
<tr>
<td>$\eta'$</td>
<td>$e^+ e^- \gamma$</td>
<td>$\pi^+ \pi^- \gamma$</td>
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</tr>
<tr>
<td>$\rho$</td>
<td></td>
<td>$\pi^+ \pi^- \gamma$</td>
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<tr>
<td>$\omega$</td>
<td>$e^+ e^- \pi^0$</td>
<td>$\pi^+ \pi^- \gamma$</td>
<td>$\pi^+ \pi^- \pi^0$</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td></td>
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</table>
### Light mesons in CLAS experiment

<table>
<thead>
<tr>
<th></th>
<th>(e^+ e^-\gamma)</th>
<th>(\pi^+ \pi^-\gamma)</th>
<th>(\pi^+\pi^-\pi^0,)</th>
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<tr>
<td>(\pi)</td>
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<td>(\eta)</td>
<td>(e^+ e^-\gamma)</td>
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</tr>
<tr>
<td>(\eta')</td>
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<td>(\pi^+\pi^-\pi^0,)</td>
<td>(\pi^+\pi^-\eta,)</td>
</tr>
<tr>
<td>(\rho)</td>
<td></td>
<td>(\pi^+ \pi^-\gamma)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\omega)</td>
<td>(e^+ e^-\pi^0)</td>
<td>(\pi^+ \pi^-\gamma)</td>
<td>(\pi^+\pi^-\pi^0)</td>
<td></td>
</tr>
<tr>
<td>(\varphi)</td>
<td></td>
<td></td>
<td>(\pi^+\pi^-\pi^0)</td>
<td>(\pi^+\pi^-\eta)</td>
</tr>
</tbody>
</table>

M. J. Amaryan et al.
CLAS Analysis Proposal, (2014)

P. Guo, I.D., A. Szczepaniak, …
First principle constraints

Unitarity: for small $s$ unitarily is "simple"

$$H(s, t) = \sum_{J}^{\infty} (2J + 1) P_{J}(z) f_{J}(s)$$

$$\text{Disc } f_{J}(s) = \rho(s) f_{J}(s+) f_{J}(s-)$$
First principle constraints

Unitarity: for small $s$ unitarily is “simple”

Analyticity
- relates scattering amplitude at different energies

\[ H(s, t) = \sum_{J}^{\infty} (2J + 1) P_J(z) f_J(s) \]

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$$f_{J}(s) = \frac{1}{2\pi i} \int_{C} ds' \frac{f_{J}(s')}{s' - s} = \int_{-\infty}^{0} \frac{ds'}{\pi} \frac{\text{Disc } f_{J}(s')}{s' - s} + \int_{4m^{2}}^{\infty} \frac{ds'}{\pi} \frac{\text{Disc } f_{J}(s')}{s' - s}$$
First principle constraints

Unitarity: for small s unitarily is “simple”

Analyticity
- relates scattering amplitude at different energies

Crossing symmetry
- the same function $H(s, t)$ should describe different processes (rotate the diagram by $90^\circ$)

\[
H(s, t) = \sum_{J}^{\infty} (2J + 1) P_J(z) f_J(s)
\]

\[
\text{Disc } f_J(s) = \rho(s) f_J(s+) f_J(s-)
\]

\[
f_J(s) = \frac{1}{2\pi i} \int_{C} ds' \frac{f_J(s')}{s' - s} = \int_{-\infty}^{0} ds' \frac{\text{Disc } f_J(s')}{\pi} \frac{1}{s' - s} + \int_{4m^2}^{\infty} ds' \frac{\text{Disc } f_J(s')}{\pi} \frac{1}{s' - s}
\]
First principle constraints

\[ \mathcal{L}_{QCD} = \sum_{f=u,d,s,c,b,t} \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f - \frac{1}{4} G^{(a)}_{\mu\nu} G^{(a)\mu\nu} \]

- at high energies: asymptotic freedom \( \rightarrow \) perturbative QCD
- at low energies: chiral symmetry

\[ SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \]

Chiral perturbation theory (\( \chiPT \))
- d.o.f. - hadrons
- expansion in mass and momenta
- Unknown coupling constants (\( L_i \)) fitted to the data

Wolfram Weise

Weinberg, Gasser & Leutwyler
First principle constraints

Unitarity

Analyticity, Crossing symmetry

QCD constraints
In practice rigorous implementation of these principles is very hard. However, for a given reaction it is possible to kinematically isolate regions where specific processes dominate.

$\pi\pi$, $\pi K$, $\pi N$ scattering Roy-Steiner, ...
Ananthanarayan et al. (2001), R. Garcia-Martin (2011),
Buttiker et al. (2001), Ditsche et al. (2012) ...
First principle constraints

In practice rigorous implementation of these principles is very hard. However, for a given reaction it is possible to kinematically isolate regions where specific processes dominate.

ππ, πK, πN scattering Roy-Steiner, …

talk of J. Ruiz de Elvira
Typical Data analysis

\[ \omega/\phi \rightarrow 3\pi \]

3 BW + background term
Typical Data analysis

\[ \omega/\phi \to 3\pi \]

Shortcomings: unitarity is not satisfied

In view of the upcoming high statistic data (CLAS, KLOE, ..)
\[ \rightarrow \] precision amplitude analysis is needed

Need to take into account final state interactions in a systematic way
(Generalized) isobar decomposition

$\omega/\phi$ is spin 1 particle:

$$H = i \epsilon_{\mu\nu\alpha\beta} \epsilon^{\mu}(p, \lambda) p_1^\nu p_2^\alpha p_3^\beta F(s, t, u)$$

$$= \sum_{J=1,3,...}^{\infty} (2J + 1) d^J_{\lambda_0}(\theta_s) f^J(s)$$
\( \text{(Generalized) isobar decomposition} \)

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\]

p.w. expansion for \( F(s,t,u) \)

\[
F(s, t, u) = \sum_{J=1,3,\ldots}^{\infty} (p(s) q(s))^{J-1} P^J_{J'}(z_s) F_J(s)
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(Generalized) isobar decomposition

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Niecknig, Kubis Schneider (2012)
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\[
\sum_{J}^{\infty} \rightarrow \sum_{J}^{J_{max}}
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\[ H = i \epsilon_{\mu\nu\alpha\beta} \epsilon^{\mu}(p, \lambda) \ p_{1}^{\nu} \ p_{2}^{\alpha} \ p_{3}^{\beta} \ F(s, t, u) \]
\[ = \sum_{J=1,3,\ldots}^{\infty} (2J + 1) \ d_{\lambda 0}^{J}(\theta_{s}) \ f^{J}(s) \]

\[ p.w. \ expansion \ for \ \mathcal{F}(s,t,u) \]
\[ \mathcal{F}(s, t, u) = \sum_{J=1,3,\ldots}^{\infty} (p(s) q(s))^{J-1} P_{J}(z_{s}) \ F_{J}(s) \]

Truncate the partial waves

So-called reconstruction theorem:

\[ \mathcal{F}(s, t, u) = \sum_{J=1,3,\ldots}^{J_{\text{max}}} \ldots \mathcal{F}_{J}(s) + \sum_{J=1,3,\ldots}^{J_{\text{max}}} \ldots \mathcal{F}_{J}(t) + \sum_{J=1,3,\ldots}^{J_{\text{max}}} \ldots \mathcal{F}_{J}(u) \]

So-called reconstruction theorem:

\[ \pi\pi \ scattering \]

Fuchs, Sazdjian, Stern (1993)
(Generalized) isobar decomposition

\[ \omega/\phi \text{ is spin 1 particle:} \]

\[ H = i \epsilon_{\mu \nu \alpha \beta} e^{\mu}(p, \lambda) p_1^{\nu} p_2^{\alpha} p_3^{\beta} F(s, t, u) \]

\[ = \sum_{J=1,3,...}^{\infty} (2J + 1) d^J_{\lambda \theta}(\theta_s) f^J(s) \]

p.w. expansion for \( F(s,t,u) \)

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Truncate the partial waves

Fuchs, Sazdjian, Stern (1993)

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Consider $J=1$ only
Discontinuity relations

Consider J=1 only

\[
\begin{align*}
\text{Disc } F(s) & = t^*(s) \rho(s) F(s) \\
\end{align*}
\]
Discontinuity relations

Consider $J=1$ only

$$\text{Disc } F(s) = t^*(s) \rho(s) F(s)$$
Consider $J=1$ only

\[ \text{Disc } F(s) = t^*(s) \rho(s) F(s) \]

\[ \text{Disc } F(s) = t^*(s) \rho(s) \left( F(s) + \hat{F}(s) \right) \]

\[ \hat{F}(s) = 3 \int_{-1}^{+1} \frac{d \cos \theta}{2} \left( 1 - \cos^2 \theta \right) F(t) \]

Khuri, Treiman (1960)
Aitchison (1977)
Discontinuity relations

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\[ \int_{-1}^{+1} d\cos \theta \rightarrow \int_{t-(s)}^{t+(s)} dt \]

\[ M_V^2 \rightarrow M_V^2 + i \epsilon \]

Bronzan, Kacser (1963)
Integral equation

\[
F(s) = \int_{4m^2}^{\infty} \frac{ds'}{\pi} \text{Disc } F(s') \frac{\text{Disc } F(s')}{s' - s}
\]

\[
\text{Disc } F(s) = t^*(s) \rho(s) \left( F(s) + \hat{F}(s) \right) + \text{inelastic } \theta(s > s_i)
\]
Integral equation

\[ F(s) = \int_0^\infty \frac{d\sigma}{4m^2} \frac{\text{Disc } F(s')}{\pi(s' - s)} \]

Disc \( F(s) = t^*(s) \rho(s) \left( F(s) + \hat{F}(s) \right) + \text{inelastic } \theta(s > s_i) \)

For practical reason we decompose

\[ F(s) = \Omega(s) G(s) \]

Disc \( \Omega(s) = \rho(s) t^*(s) \Omega(s) + \text{inelastic } \theta(s > s_i) \)

\textbf{Omnes (1958)}
Dispersion relation

Integral equation

\[ F(s) = \int_{4m^2_{\pi}}^{\infty} \frac{ds'}{\pi} \frac{\text{Disc } F(s')}{s' - s} \]

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\[ \text{Disc } \Omega(s) = \rho(s) t^*(s) \Omega(s) + \text{inelastic } \theta(s > s_i) \]

parametrize with conformal mapping expansion

Solve integral equation for \( G(s) \)

\[ G(s) = \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\text{Disc } G(s')}{s' - s} = \int_{4m_\pi^2}^{s_i} \ldots + \int_{s_i}^{\infty} \ldots \]

\[ \text{Disc } G(s) = \frac{\rho(s) t^*(s) \hat{F}(s)}{\Omega^*(s)} + \text{inelastic } \theta(s > s_i) \]
Integral equation: \[ G(s) = \int_{4m_{\pi}^2}^{\infty} \frac{ds'}{\pi} \frac{\text{Disc} G(s')}{s' - s} = \int_{4m_{\pi}^2}^{s_0} \frac{ds'}{\pi} \frac{\text{Disc} G(s')}{s' - s} + \sum_{k=0}^{\infty} a_k (\omega(s))^k \]
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\( w(s) \) is the **conformal map of inelastic contributions**

\[ \omega(s) = \frac{\sqrt{s_i} - \sqrt{s_i - s}}{\sqrt{s_i} + \sqrt{s_i - s}} \]

\[ s_i = 1 \text{ GeV}^2 \]

Yndurain (2002)
Dispersion relation

Integral equation:

\[ G(s) = \int_{4m^2/\pi}^{\infty} \frac{ds'}{\pi} \text{Disc} G(s') = \int_{4m^2/\pi}^{s_i} \frac{ds'}{\pi} \text{Disc} G(s') + \sum_{k=0}^{\infty} a_k (\omega(s))^k \]

\[ G(s) = \sum_{k=0}^{n-1} \alpha_k s^k + \frac{s^n}{\pi} \int_{4m^2/\pi}^{\infty} \frac{ds'}{s'^m} \text{Disc} G(s') \]

different from
Niecknig et. al. (2012)
Anisovich et. al. (1998)
Dispersion relation

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Benefits:

- **no need** to use approximations about unknown high energy region
- higher partial waves **does not** requires more and more coefficients (as oppose to subtraction procedure)
- coefficients are real
**Dalitz plots**

\[ \phi \to 3\pi \]

\[ \omega \to 3\pi \]

\[ \frac{d^2\Gamma}{ds \, dt} \propto |\vec{p}_+ \times \vec{p}_-|^2 |F(s, t)|^2 \]

- **Only one parameter** (overall normalization) → fixed from \( \Gamma_{\exp}(\omega/\phi \to 3\pi) \)
- **\( \phi \to 3\pi \)**: distribution clearly shows \( \rho \)-meson resonances
- **\( \omega \to 3\pi \)**: distribution is relatively flat

**KLOE (2003)**

Waiting for data ... (KLOE policy board)

Upcoming data from CLAS
On going fitting of CLAS g12 data

- Data: g12 experiment
- Reaction: $\gamma p \rightarrow p \omega \rightarrow p \pi^+ \pi^- \pi^0$
- Incoming photon energies:
  - 1.1 - 3.8 GeV (Florida group: Volker Crede, Chris Zeoli, ...)
  - >3.6 GeV (JLab: Carlos Salgado, ...)
- Files: data, reconstructed Monte Carlo, generated Monte Carlo
- 4-vector format: $<\pi^+ 4$-vec: Px, Py, Pz, E;>, ...
  incl: Q-value: likelihood for event being signal
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talk of Andrea Celentano:
  data: g11
  model: Veneziano
$\omega/\phi \rightarrow 3\pi$

Effect on the Dalitz plot

$\omega \rightarrow 3\pi$: $\sim 7\%$

$\phi \rightarrow 3\pi$: $\sim 48\%$
Dalitz plot parameters: $\omega \rightarrow 3\pi$

$$|F|^2 \propto 1 + 2\alpha z + 2\beta z^{3/2}\sin(3\phi) + 2\gamma z^2 + 2\delta z^{5/2}\sin(3\phi) + \mathcal{O}(z^3)$$

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<tr>
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<th>$\alpha \times 10^3$</th>
<th>$\beta \times 10^3$</th>
<th>$\gamma \times 10^3$</th>
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Fundamental principles (unitarity, analyticity and crossing symmetry) are very important!

3b effects are not negligible!

Upcoming high statistic data from CLAS g12!

Extend to more complicated cases like: $J^P$(arbitrary spin) $\rightarrow 3\pi$, $N^* \rightarrow NN\pi$, $D \rightarrow K\pi\pi$, ...

Thank you for your attention!