Paving the way for $\gamma p \rightarrow K^+ K^- p$
partial wave analysis: KN scattering

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Interesting channels

\[ X = \pi \pi, K \bar{K}, \pi \eta, \pi \eta' \]

\[ X = \rho, \rho_3, \phi, \phi_3, f_0, f_2, f_2', a_0, a_2, \pi_1 \]

Glueballs?

Hybrid?
\( \gamma p \rightarrow K^+K^-p \)

Amplitude depends on 5 Mandelstam variables:
- One is fixed: \( s \)
- Two are measured: \( s_{K^+K^-} \) and \( s_{K^-p} \)
- Two are integrated out: \( t_{\gamma K^+} \) and \( t_{pp'} \)
K⁺K⁻ Dalitz plot

\[ W = 5 \text{ GeV} \]

Phi mesons

Double Regge

Hyperons

Disclaimer: not actual data
(actual data from g12 CLAS@JLab close to completion)
Things we expect/hope to study

- **Exotics**
  - new physics that will help us understand the role of the gluon and confinement

- **Strangeonia** \((s\bar{s})\)
  - this spectrum is not well studied and looks pretty empty. Also information on gluons and virtual quarks

- **Hyperons**
  - Strange content of hadrons
$K^+K^-$ Dalitz plot

$W = 5$ GeV

Hyperon

Meson

Double Regge
Deck Model: one step at a time

Reggeon

X
KN scattering

Resonance region

High energy region
KN model

- Resonance region: K-matrix, coupled channels
- High energy: Regge
- Connection through dispersion theory and Finite Energy Sum Rules
KN in resonance region

- Partial-wave analysis ($L_{\text{max}} = 5$)

\[ t^I(s, \cos \theta) = \sum_{\ell} f_{\ell}^I(s) P_{\ell}(\cos \theta) \]

- Coupled channels
- Unitarity
- Analyticity
- Right threshold behavior (angular momentum barrier)
- Resonances and backgrounds are incorporated “by-hand” through K matrices
Unitarity

\[
S_\ell = I + 2i \left[ C_\ell(s) \right]^{1/2} T_\ell(s) \left[ C_\ell(s) \right]^{1/2}
\]

\[
\text{Disc } T_\ell(s) = 2i \text{ Im}T_\ell(s) = 2i \ T_\ell^\dagger C_\ell(s)T_\ell(s)
\]

\[
T_\ell(s) = K(s)^{-1} - i \rho_\ell(s)
\]

\[
[i \rho_\ell(s)]_{kk} = \frac{s - s_k}{\pi} \int_{s_k}^{\infty} \frac{[C_\ell(s')]_{kk}}{s' - s} ds'
\]

\[
k = \pi \Sigma, \bar{K}N, \pi \Lambda, \pi \Sigma(1385), \pi \Lambda(1520), \eta \Sigma, \eta \Lambda, \bar{K}^* N, \pi \Delta(1232), \pi \pi \Sigma, \pi \pi \Lambda
\]
Phase space/analyticity

\[
[C_{\ell}(s)]_{kk} = \frac{q_k(s)}{q_0} \left[ \frac{q_k^2(s) r^2}{1 + q_k^2(s) r^2} \right] \ell
\]

- Right threshold behavior
  - Angular momentum barrier
- Right high-energy behavior
- \( r = 1 \) fm (interaction radius)

\[
[i \rho_{\ell}(s)]_{kk} = \frac{s - s_k}{\pi} \int_{s_k}^{\infty} \frac{[C_{\ell}(s')]_{kk}}{s' - s} \frac{ds'}{s' - s_k} = -a_0 \frac{a^\ell}{\pi \Gamma(\ell)} \left[ \frac{\pi \Gamma(\ell)(s - s_k) \sqrt{s_k - s}}{1 + a(s - s_k)} \right.
\]
\[
- \frac{\sqrt{\pi} \Gamma(\ell + 1/2)}{\ell a^{\ell+1/2}} ([1 + a(s - s_k)]_{2F1}[1, \ell + 1/2, -1/2, 1/a(s_k - s)]
\]
\[
- [3 + 2\ell + a(s - s_k)]_{2F1}[1, \ell + 1/2, 1/2, 1/a(s_k - s)]
\]

Valid for \( \ell \) real and bigger than -1/2
K matrix

**Resonance**

\[
[K_a(s)]_{kj} = x^a_k \frac{M_a}{M^2_a - s} x^a_j
\]

Generates pole in the 2nd Riemann sheet

**Background**

\[
[K_b(s)]_{kj} = x^b_k \frac{M_b}{M^2_b + s} x^b_j
\]

Generates pole in the real axis for s<0 in the 1st Riemann sheet

The background behaves as a smooth function in the physical region
Analytical structure
The amount of Riemann sheets depends on the number of open channels: $2^N$
Coupled channels: 1 amplitude

\[
[T(s)]_{kj} = x_k^a \, T_a(s) \, x_j^a
\]

\[
T_a(s) = \frac{M_a}{M_a^2 - r_a s - i M_a \Sigma_a(s, \ell)}
\]

\[
[x_k^a]^2 = \frac{[y_k^a]^2}{[C \ell(M_a^2)]_{kk}}
\]

\[
\Sigma_a(s, \ell) = \sum_{k=1}^{n_C} \Sigma_k^a(s, \ell) = \sum_{k=1}^{n_C} [\rho_\ell(s)]_{kk} \, [x_k^a]^2
\]

\[
\Gamma_a(s) = \sum_{k=1}^{n_C} \Gamma_k^a(s) = \sum_{k=1}^{n_C} \theta (M_a^2 - s_k) \, \text{Im} \, \Sigma_k^a(s)
\]

\[
\Gamma_a = \sum_{k=1}^{n_C} \Gamma_k^a = \sum_{k=1}^{n_C} [y_k^a]^2 \, \theta (M_a^2 - s_k)
\]
Coupled channels: 2 amplitudes

\[
[K(s)]_{kj} = x_k^a K_a(s) x_j^a + x_k^b K_b(s) x_j^b
\]

\[
[T(s)]_{kj} = \frac{1}{D_2(s)} \left[ x_k^a c_{aa}(s) x_j^a + x_k^a c_{ab}(s) x_j^b + x_k^b c_{ba}(s) x_j^a + x_k^b c_{bb}(s) x_j^b \right]
\]

\[
c_{aa}(s) = T_a(s)
\]

\[
c_{bb}(s) = T_b(s)
\]

\[
c_{ab}(s) = c_{ba}(s) = i\epsilon_{ab}(s)T_a(s)T_b(s)
\]

\[
D_2(s) = 1 + [\epsilon_{ab}(s)]^2 T_a(s)T_b(s)
\]

\[
\epsilon_{ab}(s) = \epsilon_{ba}(s) = \sum_{k=1}^{n_C} [\rho_\ell(s)]_{kk} x_k^a x_k^b
\]

\[
\epsilon_{aa}(s) = \sum_{k=1}^{n_C} [\rho_\ell(s)]_{kk} [x_k^a]^2 = \Sigma_a(s, \ell)
\]
Coupled channels: General case

\[
[K(s)]_{kj} = \sum_{a} x_{k}^{a} K_{a}(s) x_{j}^{a}
\]

\[
[T_{\ell}(s)]_{kj} = \frac{1}{D(s)} \sum_{a,b} x_{k}^{a} c_{ab}(s) x_{j}^{b}
\]

\[
T_{\ell}(s) = \left[ K^{-1}(s) - i\rho_{\ell}(s) \right]^{-1}
\]

Solved up to six K matrices to obtain the \( c_{ab}(s) \) and \( D(s) \)
Fit coupled-channel single-energy partial waves from Kent State University analysis of processes:

\[
\begin{align*}
\bar{K}N &\rightarrow \bar{K}N \\
\bar{K}N &\rightarrow \pi\Sigma \\
\bar{K}N &\rightarrow \pi\Lambda
\end{align*}
\]

Zhang et al., PRC88 (2013) 035204

Caveat: We lose correlations among partial waves

Our objective is to have a reasonable KN model

Model can be readjusted once we extend to the two kaon photoproduction process, getting more insight on hyperons
D_{15}

- 2 “resonances” and 1 “background”
- Poles in the nearby unphysical sheet
- $\chi^2$/dof=1.15
- Well-constrained pole corresponds to $\Sigma(1775)$ in PDG
- Errors computed through bootstrap technique
D_{13}

- 1 “resonance” and 2 “backgrounds”
- $\chi^2$/dof=0.92
- Pole corresponds to $\Sigma(1670)$ in PDG
- Pole at -0.40 GeV$^2$ in 1st RS due to the background amplitudes
$P_{03}$

- 2 “resonances” and 2 “backgrounds”
- $\chi^2$/dof=1.65
- Closest pole to physical axis corresponds to $\Lambda(1890)$ in PDG
- Pole at 0.08 GeV$^2$ in 1st RS due to the background amplitudes
KN scattering in high energy region

V. Mathieu’s talk yesterday
Connecting high and low energies

- Let’s use πN scattering as a playground
- Dispersion theory (Finite energy sum rules)
- We use high-energy to constrain low-energy
- Construct Im A from 0 to infinity via FESR
- Reconstruct amplitude from dispersion relations

V. Mathieu’s talk yesterday
Summary

This is as far as I can go...

- We have a model for the resonance region with the right properties
- We are finishing the fits
- We are finishing the high-energy Regge as well as the pion-nucleon case
- Next steps will be to connect high and low energy in KN, build the full two kaon photoproduction amplitude and compare to data