One or Two $a_{-1}$ Axial Vector Mesons?


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Based on:
“The twofold emergence of the $a_{-1}$ axial vector meson in high energy hadronic production” [arXiv:1501.04643]
and
“Unitary coupled-channel analysis of diffractive production of the $a_{-1}$ resonance”
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both by
Jean-Louis Basdevant and Edmond L Berger
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Overview

• COMPASS: data of unprecedented statistical power, coupled with tour-de-force analysis: evidence for an axial vector $J^{PC} = 1^{++}$ peak in the P wave $\pi f_0(980)$ channel at about 1420 MeV.

• The usual $a_1$ is in the S-wave $\pi \rho$ channel at about 1260 MeV.

  - Are there two $a_1$ so close in mass or is the P wave $\pi f_0$ another decay mode of the usual $a_1$? If so, why at a different mass?

  - Revive unitary coupled-channel research done in 1975 - 1979 on $\pi \pi \pi$, $\pi K \bar{K}$; this time with both S and P wave channels

• Describe here the new study we did recently.

• Conclusion: One $a_1$ suffices to explain the two peaks.
Outline

- Deck mechanism for production of non-resonant $\rho\pi$ and $f_0\pi$ in the mass range $M = 1$ to $2$ GeV
- Incorporation of resonant behavior $a_1 \rightarrow f_0\pi$, $a_1 \rightarrow \rho\pi$

One resonance.

- Multichannel final state unitarization: — two channel, strong interaction S matrix, …… reaction amplitude that includes both Deck “background” and resonance

- Results: (a) separate mass peaks in $J^{PC} = 1^{++}$ S wave $\rho\pi$ and P wave $f_0\pi$ channels, and (b) relative phase between the two amplitudes — consistent with data
Deck Production Mechanism

- Consider $\pi p \rightarrow \pi \pi \pi p$ at large incident $\pi$ momentum and small momentum transfer to the target.

- Think of the 3 pion system as a superposition of quasi-two-body systems $(\pi \pi \pi) \rightarrow \pi \rho, \pi f_0, \ldots$.

- One pion exchange production, followed by diffractive scattering of the virtual pion from the target:
Deck Amplitude - 1

- Deck amplitude for $\pi p \rightarrow \rho\pi p$

$$T_D^\rho = g_{\rho\pi\pi} K_\rho(t_2) \frac{1}{m_\pi^2 - t_2} i s_{13} e^{b t_1} \sigma_{\pi p}$$

- Similar expression for $\pi p \rightarrow f_0\pi p$

- In the $\rho\pi$ or $f_0\pi$ rest frame, the Deck amplitude contributes to several partial waves.

- For $J^{PC} = 1^{++}$ one must project the S wave component for $\rho\pi$ and the P wave component for $f_0\pi$

- Re-express the invariants $s_{13}$ and $t_2$ in terms of t-channel angles

$$t_2 = g_1(M, t_1) + g_2(M, t_1) \cos \theta_t$$
$$s_{13} = g_3(s, M) + g_4(s, M, t_1) \cos \theta_t + g_5(s, M, t_1) \sin \theta_t \cos \phi_t$$

- Deck amplitude is a rational function so one can project analytically all partial waves S, P, D, ... (all m), for any value of $t_1$
Deck Amplitude - 2

- Consider an expansion for small $t_1$ (where the data are concentrated) of the partial wave projections of the Deck production amplitude. Define expansion parameter

$$\Theta_1 = \frac{t_1}{(M^2 - m_{\pi}^2)}.$$

- The S-wave projection is

$$T_{S, Deck}^T = -\frac{s}{(M^2 - m_{\pi}^2)} \times \left(1 - \frac{1}{2} \Theta_1 \left(\frac{3M^2 + m_{\pi}^2}{(M^2 - m_{\pi}^2)} - \frac{E_0}{E_{\pi}}\right) \left(\frac{1}{y} \ln \left(\frac{1 + y}{1 - y}\right)\right)\right)$$

- The P-wave projection is

$$T_{P, Deck}^T = +\frac{3}{2} \frac{s}{(M^2 - m_{\pi}^2)} \Theta_1 \times \left(\frac{(3M^2 + m_{\pi}^2)}{(M^2 - m_{\pi}^2)} - \frac{E_0}{E_{\pi}}\right) \left(\frac{-2}{y} + \frac{1}{y^2} \ln \left(\frac{1 + y}{1 - y}\right)\right)$$

- The P-wave projection vanishes at $t_1 = 0$; more importantly, it passes through 0 for a special value of $M$ — sign change
• P wave amplitude crosses zero near $M \simeq 1.38$ GeV

• This sign change drives the relative phase change between the P wave and other waves

• P wave intensity is also much smaller ($\sim 10^{-3}$) than the S wave
Unitarization - 1

- Deck amplitude $T^{Deck}$ produces $J^{PC} = 1^{++}$ non resonant enhancements near threshold in $\rho\pi$ and $f_0\pi$.

- The $\rho\pi$ and $f_0\pi$ are strongly interacting systems. They interact in the final state, even in the one channel $\rho\pi$ case. Final state interactions must be included. They are inevitable and non-negligible if there is a resonance, such as the $J^{PC} = 1^{++} q\bar{q}$ state of the quark model.

- Construct a full amplitude $T^{u}_{Deck}$ that includes final state interactions, the $q\bar{q}$ state, and respects unitarity (no double counting).
Unitarization - 2

• Impose unitarity by requiring that the amplitude $T^u_D(M)$ satisfies proper discontinuity relations.

• $T^u_D(M)$ has a right-hand unitarity discontinuity starting at the lowest threshold, $M = m_\rho + m_\pi$

• $T^+$ is its value above the cut; $T^-$ is the value below the cut.

• Unitarity relationship $T^+ = ST^- $; $S$ is the strong interaction unitary $S$ matrix that describes

\[ \rho\pi \rightarrow \rho\pi, f_0\pi \rightarrow f_0\pi, \rho\pi \rightarrow f_0\pi \]

• Aside: we do not know $S$. Parametrize it in terms of a K matrix and determine the parameters by comparing with data.
Unitarization - 3

- Theory task: Construct an analytic and unitary $T^u_D$ from knowledge of its singularities: (a) right hand unitarity discontinuities; and (b) “left-hand” pole singularity supplied by the Deck production amplitude, $T^{-1}_D \sim (M^2 - m^2_{\pi})$.

- Solution in terms of an analytic 2 X 2 $D(M^2)$ matrix that has only a right hand unitarity discontinuity: $D^+(M) = SD^-(M)$.

- Dispersion integral leads to

$$T^u_D(M^2) = T_D(M^2) - \frac{1}{\pi} D(M^2) \times \int_{(m_\rho + m_\pi)^2}^{\infty} ds' \\frac{ImD(s')T_D(s')}{(s' - M^2)} .$$  \hspace{1cm} (1)

- Expression is our Deck amplitude with resonant final state interactions taken into account.

- Properties: (a) same left-hand production singularity as $T_{Deck}$; (b) satisfies unitarity; (c) reduces to $T_{Deck}$ if no rescattering.
**Unitarization - “Practical” details**

- Parametrize the coupled channel S matrix in terms of a K matrix:

\[
K(M^2) = \begin{pmatrix}
\frac{g_1^2}{s_1 - M^2} & \frac{g_1 g_2}{s_1 - M^2} \\
\frac{g_1 g_2}{s_1 - M^2} & \frac{g_2^2}{s_1 - M^2}
\end{pmatrix}.
\]

- Simple pole parametrization yields analytic expression for D matrix. \( g_1, g_2 \) are coupling strengths to the two channels.

\[
D(M^2) = \frac{1}{\mathcal{D}_0(M^2)} \begin{pmatrix}
g_1 & -g_2(s_1 - M^2 - \alpha^2 C_2) \\
g_2 & g_1(s_1 - M^2 - \alpha^2 C_1)
\end{pmatrix}
\]

- The denominator \( \mathcal{D}_0(M^2) = (s_1 - M^2 - g_1^2 C_1(M^2) - g_2^2 C_2(M^2)) \) has the appearance of a resonance factor. In the one channel case \( \mathcal{D}_0^{-1}(M^2) \sim e^{i\delta} \sin \delta \)

- \( \alpha^2 = g_1^2 + g_2^2; \) \( C_1 \) and \( C_2 \) are Chew-Mandlestam functions
Behavior of $D(M)$
Direct Production Term

- In addition to its affects manifest in the unitarized Deck amplitude, the resonance may be produced directly via a diffractive coupling, $\pi p \rightarrow a_1 p$

\[ T_{dir}(s, M^2) = \frac{i s \sigma_{\pi p} G}{D_0(m^2)} \left( \begin{array}{c} f_1 \\ f_2 \end{array} \right), \]

- Those acquainted with the study of $\pi\pi$ scattering in photo-production, $\gamma p \rightarrow \pi\pi N$, will recognize this term as the analog of the “vector- dominance” term in $\rho$ production; the Deck term in the photo production case plays a role in modifying the $\rho$ line shape (e.g., Paul Soding, 1966).
Final Amplitude and Parameters

\[ T(M^2) = T_D^u(M^2) + T_{dir}(M^2) \]  \hspace{1cm} (1)

- Recall that $T(M^2)$ is a two-dimensional vector; upper and lower components for $\rho\pi$ and $f_0\pi$, respectively.

- Parameters are the K matrix pole position $s_1$ and the pole coupling strengths $g_1$ and $g_2$.

- Plus the two coupling strengths in the “direct” term, $Gf_1$ and $Gf_2$. 
Comparison with data

- We have not made a $\chi^2$ fit.
- Focus on the momentum transfer $t$ interval $[0.10 \text{ to } 0.13] \text{ GeV}^2$
- Trial and error: find appropriate values of the $a_1$ mass and width (defined by the position of the pole on the second sheet) that give the observed mass peaks. Obtain:
  
  $M(a_1) \simeq 1.40 \pm 0.02 \text{ GeV},$
  
  $\Gamma(a_1) \simeq 0.30 \pm 0.05 \text{ GeV}.$
- These values fix $s_1 \sim 2.002 \text{ GeV}^2$; $g_1 \sim 0.732 \text{ GeV}$.
- The ratio $\gamma = g_2/g_1$ was varied to give the observed relative intensity of the two peaks: central value $\gamma = g_2/g_1 = -0.08$.
- Determine the amount of “direct” production by placing the two peaks at the desired locations:
  
  $G\sigma_{\pi p}f_1 = 120; \ G\sigma_{\pi p}f_2 = 5.5$
\[ J^{PC} = 1^{++} \rho \pi \] **mass distribution**

- Note that unitarization sharpens the Deck amplitude
- Overall peak location at about 1260 MeV, width about 280 MeV
- The peak does not have a symmetric Breit-Wigner form
\[ J^{PC} = 1^{++} f_0 \pi \] mass distribution

- Deck in \( f_0 \pi \) is narrow and very near threshold
- The final peak is pushed higher in mass, close to 1420 MeV; width about 140 MeV
- Note the second peak in \( f_0 \pi \) predicted just below 1200 MeV
Both on the same figure

- Scale up the $f_0 \pi$ distribution by X 650
• Curves showing the relative phase as a function of $M$ for three choices of the ratio of coupling strengths.

• Sharp rise of the relative phase related to the zero in the P wave production amplitude.
Outlook and perspectives

- Main features of the COMPASS data, two mass peaks separated by about 160 GeV, with pronounced relative phase motion, are compatible with a single $a_1$

- New determination of the mass and width of the $a_1$ along with its branching fraction into $f_0 \pi$

- Rediscovered in this example that, although a peak is often associated with a resonance, its precise mass and width depend also on the dynamics of the mechanism by which it is produced.

- Here, the same Deck production mechanism has very different character in the S-wave and P-wave channels, leading to a shift by about 160 MeV in the observed positions of the $J^{PC} = 1^{++}$ state.

- If one could do low-energy $\rho \pi$ and $f_0 \pi$ elastic scattering, one would observe a single resonance peak with mass and width $M \sim 1.36$ GeV and $\Gamma \sim 0.31$ GeV
Dependence on momentum transfer

- We have results for arbitrary values of the momentum transfer to the target, $t_1$; the changes in mass spectra and phases are modest. Paper in preparation.

- What about the differential cross section as a function of $t_1$?

- Recall: the final amplitude is a sum of two terms:

$$T(M^2, t_1) = T_D^u(M^2, t_1) + T_{dir}(M^2, t_1).$$

- Each term has its own $t_1$ dependence properties; the direct term has the same $t_1$ dependence for both channels.

- However, there is a (well known) strong mass dependence of the $t_1$ distribution for the Deck term, both in theory and experiment. The slope is considerably steeper at low $M$ than at higher $M$. Moreover, there is the kinematic suppression at small $t_1$ for the P-wave channel.
Other channels

- $\pi_2(1670)$ and $\pi'_2$ as S- and D-waves of $f_0\pi$