Application of the Veneziano model to the light meson decays

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Introduction

1. Veneziano model
2. Light meson decays
3. $\omega \rightarrow 3\pi$ analysis
4. Other channels
5. Conclusions
The original Veneziano amplitude

**Veneziano amplitude for 4-legs reactions \((B_4)\):**

- Originally developed\(^1\) for the reaction \(\omega \pi \rightarrow \pi \pi\)
- “Compact” expression for the full amplitude

\[
A(s, t, u) = \frac{\Gamma(1 - \alpha(s))\Gamma(1 - \alpha(t))}{\Gamma(2 - \alpha(s) - \alpha(t))} + (s, u) + (t, u)
\]

\(\alpha(s):\) Regge Trajectory

**Main properties:**

- **Analyticity.** Singularities are:
  - **Poles** for negative arguments of Gamma function - double-poles are canceled by the denominator.
  - Singularities of the Regge trajectory (typically cuts from \(\Im(\alpha)\))
- **Crossing symmetry**
- **Regge asymptotic behavior:** \(A(s, t) \xrightarrow{s \to \infty} \frac{1}{s} \Gamma(1 - \alpha_t)(-s)^{\alpha_t}\)

\(^1\) G. Veneziano, Nuovo Cimento A 57:190-7, 1968
Amplitude extension

The original Veneziano amplitude can be generalized as:

\[
A_{n,m}(s, t, u) = \frac{\Gamma(n - \alpha_s)\Gamma(n - \alpha_t)}{\Gamma(n + m - \alpha_s - \alpha_t)} + (s, u) + (t, u)
\]

Requirements:

- No double poles in overlapping channels: \( n \geq m \)
- Correct asymptotic behavior: \( n, m \geq 1 \)

The full reaction amplitude is a linear combination of different Veneziano terms:

\[
A(s, t, u) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} c_{n,m} A_{n,m}(s, t, u)
\]

Individual poles and trajectories are enhanced or suppressed depending on the coefficients \( c_{n,m} \).
Different authors employed the Veneziano model for the analysis of the at-rest annihilation $N\bar{N} \rightarrow 3\pi$, using a finite number of Veneziano terms.

- Lovelace\(^2\): a single term amplitude, $n = m = 1$, $\alpha_s = 0.483 + 0.885s + 0.28i\sqrt{s - 4m^2}$
- Altarelli\(^3\): 5 terms with $n + m \leq 3$ (to reproduce the zero at $\alpha_s + \alpha_t \simeq 3$)
- Gopal\(^4\): 5 terms with $n + m \leq 3$, $\alpha_s = 0.483 + 0.885s + iA(s - 4m^2)^B$, $B < 1$

\(^2\) C. Lovelace, Phys. Lett. 25B (1968), 264
\(^3\) G. Altarelli, Phys. Rev. 183 (1969), 1469
\(^4\) G. P. Gopal, Phys. Rev. D 3 (1971), 2262
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2 C. Lovelace, Phys. Lett. 25B (1968), 264
The Pennington-Szczepaniak approach

The Pennington-Szczepaniak\(^5\) model adopts a more systematic approach: construct combinations of \(A_{n,m}\) that result in an amplitude containing only a finite number of poles.

\[
A_{n,m}(s, t) = \frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n+m-\alpha_s-\alpha_t)}
\]

**Example: keeping only the pole at \(\alpha_s = 1\)**

- Only present in the \(A_{11}\) amplitude: residual determined by \(c_{11}\)
- To cancel the other \(A_{11}\) poles, choose:

\[
c_{n,1} = \frac{c_{11}}{\Gamma(n)} , \quad c_{n,2} = -\frac{c_{11}}{\Gamma(n-1)}
\]

\(c_{n,m} = 0\) for \(n \geq 3\)

Result: \(A_1(s, t) = c_{1,1} \frac{2-\alpha_s-\alpha_t}{(1-\alpha_s)(1-\alpha_t)}\)

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\(^5\) A. P. Szczepaniak and M. R. Pennington, arXiv:1403.5782
The Pennington-Sczepaniak approach

Generalization: the amplitude $A_n$ containing a pole at $\alpha_s = n$ is:

$$A_n(s, t) = \frac{2n - \alpha_s - \alpha_t}{(n - \alpha_s)(n - \alpha_t)} \sum_{i=1}^{n} a_{n,i}(-\alpha_s - \alpha_t)^{i-1}$$

Asymptotic-behavior not “a-la-Regge” any more! This requires an infinite number of poles. To fix this:

- Use $c_{n,m}$ to isolate a single pole
- Truncate the $A_{n,m}$ sum up to $n = N$, with $N \gg \alpha' E_0^2$

Result:

$$A_n(s, t, N) = \frac{2n - \alpha_s - \alpha_t}{(n - \alpha_s)(n - \alpha_t)} \sum_{i=1}^{n} a_{n,i}(-\alpha_s - \alpha_t)^{i-1} \cdot \frac{\Gamma(N + 1 - \alpha_s)\Gamma(N + 1 - \alpha_t)}{\Gamma(N + 1 - n)\Gamma(N + n + 1 - \alpha_s - \alpha_t)}$$
Within the HASPECT\textsuperscript{6} analysis group we are analyzing light-meson decays employing the PS Veneziano model. Theoretical input was provided by the JPAC group.

### Objectives

- Fit the amplitude to the data and validate it
- Verify the amplitude predictivity power by determining the parameters of known resonances (pole position - couplings)
- Give feedback to JPAC, while preparing to the $B_5$ development

### Channels

- $\omega \rightarrow \pi^+ \pi^0 \pi^-$
- $\eta' \rightarrow \pi^+ \pi^- \eta$
- $f_1(1285) \rightarrow \pi^+ \pi^- \eta$

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The general form of the $\omega \rightarrow 3\pi$ decay amplitude follows from the $\omega$ spin-1 nature:

$$F_{\lambda}(s, t, u) = \varepsilon_{\mu \nu \alpha \beta} p_+^{\nu} p_-^{\alpha} p_0^{\beta} \varepsilon^\mu_{\lambda} A(s, t, u)$$

$$I = \sum_{\lambda, \lambda'} F_{\lambda}^* \rho_{\lambda'}^\lambda F_{\lambda'} = K^2 |A|^2 W_\rho(\theta, \phi)$$

- $K$: P-wave (Kibble boundary function) = $|\vec{p}_a \times \vec{p}_b|_{\omega-frame}$
- $W_\rho$: decay plane normal vector distribution in the decay frame

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\( \omega \rightarrow 3\pi \) decay: dynamics

Bose statistics and isospin conservation require \( F \) symmetric. \( K \) is anti-symmetric: no \( \pi - \pi \) spin-even resonances are allowed.

Residue of \( A_n(s, t) \) for \( \alpha_s = n \):

\[
\text{Res}(A_n) = \sum_{i=1}^{n} a_{n,i} (-n - \alpha_t)^{i-1}
\]

Polynomial of order \( n - 1 \) in \( t \): coefficients \( a_{n,i} \) must be constrained to cancel even \( t \) powers

- \( n = 1 \): ok
- \( n = 2 \): \( a_{22} = 0 \)
- \( n = 3 \): \( a_{31} = 2a_{3,2}(3 + \alpha_0) \)
- \( \ldots \)

Normalization: from the experimental value \( \Gamma_{\omega} = 7.56 \text{ MeV} \)

\[
\frac{d\Gamma}{ds \, dt} = \frac{1}{32 (2\pi M_\omega)^3} K^2 |A|^2
\]
Data were obtained by measuring the $\omega$ meson exclusive photo-production reaction with the CLAS detector in Hall B at Jefferson Laboratory.

The g11 run period

- May-June 2004
- Tagged Bremsstrahlung photon-beam: 1.5-3.8 GeV
- 40-cm long LH$_2$ target
- Integrated luminosity $\simeq 80 \text{ pb}^{-1}$
CLAS detector in Hall B at Jefferson Laboratory: almost $4\pi$ detector optimized for multi-particle final states

- Toroidal magnetic field (6 supercond. coils)
- Drift chambers (3 layers)
- Time-of-flight counters
- Electromagnetic calorimeters
- Charged particle performances:
  - Acceptance: $8^\circ < \theta < 142^\circ$
  - Resolution: $\delta p/p \simeq 1\%$, $\delta \theta < 1 \text{ mrad}$
Data selection (B. Vernansky)

Events selection:
- 1 proton, 1 $\pi^+$, 1 $\pi^-$ measured
- $MM(p\pi\pi)<450$ MeV (loose cut around missing $\pi^0$)
- $|MM(p)-M_\omega|<150$ MeV (loose cut around $\omega$)
- After 1-C kinematic fit, keep events with $CL>0.1$

Background rejection: $Q$-value method
- Event-by-event side-band subtraction
- Each event gets an associated probability ($Q$-value) to be a signal event rather than a background event
- Discriminating variable: $3\pi$ invariant mass shape

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8 M. Williams, arXiv:0804.3382
Amplitude analysis fit procedure

Event-by-event maximum likelihood unbinned fit, performed independently in different bins of $W \equiv \sqrt{s}$, from 1.9 to 2.5 GeV:

$$\log \mathcal{L} = \sum_{i=1}^{N_{\text{data}}} \log I_i - \frac{1}{N_{\text{gen}}} \sum_{j=1}^{N_{\text{acc}}} I_j$$

The decay amplitude does not depend on the production variables $\sqrt{s}$ and $\cos(\theta_{CM})$, but the detector acceptance does: $I$ must be the intensity for the full reaction $\gamma p \rightarrow p\omega \rightarrow p3\pi$

Alternatively, weight MC events by

- $\omega$ photo-production cross-section, $\frac{d\sigma}{d\cos(\theta_{CM})}$
- Decay plane normal vector distribution, $W_\rho$

This allows to perform the fit with $I_{\text{decay}} \equiv K^2|A|^2$ only.

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9 Results from M. Williams et al, Phys. Rev. C 80, 065208 (2009)
The AmpTools software

The AmpTools package is a collection of libraries that are useful for performing unbinned maximum likelihood fits to data using a set of interfering amplitudes\textsuperscript{10}.

- Developed by H. Matevosyan, R. Mitchell, M. Shepard at Indiana University
- Freely available at http://amptools.sourceforge.net, including examples and documentation

Physics embedded in the framework: almost nothing!

\[
I = (\tau, \vec{x}) = \sum_i | \sum_j V_{i,j} A_{i,j}(\tau, \vec{x}) |^2
\]

AmpTools philosophy: provide a full set of C++ classes, that the user has to derive according to the specific application (data readout, \textbf{amplitude coding}, fit, plot).

Parallelization is available:

- GPU: CUDA language
- Multi-core: MPI

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\textsuperscript{10} See https://www.jlab.org/div_dept/theory/seminars/2012_fall/Ryan_Mitchell_Dec10.pdf for a short introduction
Results: fit with $A_1$ only

**First fit:** $A = A_1$ only ($\rho$ pole), using the “nominal” $\pi\pi$ Regge trajectory parametrization:

$$\alpha_s = 0.47 + 0.9s + 0.12i \sqrt{s - 4m^2_\pi} \rightarrow s_\rho = 0.58 - 0.095i \ (M = 0.76 \text{ GeV}, \Gamma = 0.124 \text{ GeV})$$

Fit parameters: $a_1$ (overall normalization)

**Results**

- Very good agreement data-weighted MC for production variables
- Veneziano model describes well the $\omega \rightarrow 3\pi$ decay
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**Second fit: $A = A_1 + A_2$ ($\rho$ and $\rho(1450)$ poles):**

$$\alpha_s = 0.47 + 0.9s + 0.12i \sqrt{s - 4m_{\pi}^2}$$

$$\begin{align*}
s_{\rho} &= 0.58 - 0.095i \\
s_{\rho(1450)} &= 1.69 - 0.169i
\end{align*}$$

Results

- The projection of $A_1$ and $A_2$ on CLAS is very similar
- Data has limited sensitivity to higher $\rho$ states: the contribution of the $\rho(1450)$ to the total intensity is less than 10%
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<table>
<thead>
<tr>
<th>Invariant mass after CLAS projection</th>
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<td>$\pi\pi$</td>
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Results: fit with $A_1$, free Regge parameters

Third fit: $A = A_1$ ($\rho$ pole), free parameters in the Regge trajectory

$$\alpha_s = \alpha_0 + \alpha' s + g i \sqrt{s - 4m_{\pi}^2} \rightarrow s_{\rho} : \text{Solve}(\alpha_s = 1)$$

Results

- Fit with free $\alpha_0$ or free $\alpha'$ converges for all $W$ bins:
  - Free $\alpha_0$:
    $$(0.54 \pm 0.02) \rightarrow M_{\rho} = (0.71 \pm 0.02)$$
  - Free $\alpha'$:
    $$(0.96 \pm 0.03) \rightarrow M_{\rho} = (0.72 \pm 0.01)$$
- Fit with free $g$ and ($\alpha_0$ or $\alpha'$):
  - There is no sensitivity to the width; parameters are not stable in $W$.
  - The $\rho$ mass is stable.
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η' → ηππ and f_1(1285) → ηππ (A.Rizzo)

Goals:

- Validate the PS Veneziano Model with these channels
- Fit the a_0 signal in the f_1(1285) decay: determine the resonance properties (pole position - couplings) from the amplitude
- Perform a global analysis, using the same Regge trajectory expression in the two reactions
Events selection (A. Rizzo)

Events selection ($\eta'$ case):

- $E_\gamma > E_{thr} = 1.45$ GeV
- 1 proton, $\geq 1 \pi^+ , \geq 1 \pi^-$ measured
- $|MM(p)-M_{\eta'}|<100$ MeV (cut around $\eta'$)
- Events with $2 \pi$ : cut on $MM(p\pi\pi)$ around $\eta$
- Events with $\geq 2\pi$: select “best-matching” pions and proceed as before

Background rejection: $sPlot$ method\(^\text{11}\)

- Dedicated to the exploration of data samples populated by several sources of events
- Unfold the contributions of different sources (signal-background) to the distribution of a given variable
- PDFs must be known (a part from free fit parameters)

\(^{11}\) arXiv:0602.023
Results (A. Rizzo)

Current results:

- A clean sample of $\eta' \rightarrow \eta\pi\pi$ and $f_1(1285) \rightarrow \eta\pi\pi$ events has been selected
  - $a_0$ signal is clearly visible in the $f_1(1285)$ Dalitz plot
- The Veneziano amplitude for these processes is being developed at JPAC

Next steps:

- Produce MC events for PWA (framework is ready)
- Proceed with the Maximum Likelihood fit
Conclusions

• The Pennington-Szczepaniak model is an evolved version of the original Veneziano amplitude. Each term in the amplitude contains a finite number of poles (in the available kinematic region), and has the proper asymptotic behaviour.

• The analysis of the $\omega \rightarrow 3\pi$ channel qualitatively validated the PS model.
  
  • The amplitude qualitatively reproduces the data, using the “nominal” Regge trajectory parametrization.
  • There is reduced sensitivity to states other than the $\rho(770)$:
    $I_2 \simeq 10\% I_1$
  • By freeing the parameters in the Regge trajectory parametrization, it is possible to determine the mass of the $\rho$ meson - although result is smaller than the nominal value

• The channels $\eta' \rightarrow \eta\pi\pi$ and $f_1 \rightarrow \eta\pi\pi$ will be a further benchmark for the amplitude.
  
  • The $f_1$ will permit to probe higher trajectories, due to the larger available phase-space