Compton Source of Twisted Photons

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Questions Addressed

- How do optical vortices excite atoms (and nuclei)?
  - Are there any differences in excitation of higher-angular momentum states (compared to plane waves)?
  - Are (Orbital Angular Momentum) OAM beams more efficient for exciting high OAM?
Introduction

- Photons carry linear momentum \( p = \hbar k \) \((k=\text{wave vector})\)
- Photons carry both spin angular momentum (SAM) and orbital angular angular momentum (OAM) – may be separated in paraxial approximation
  - Circularly polarized plane-wave photons carry \( J_z = \pm \hbar \) along the propagation direction \( z \) (Beth’s experiment, 1936)
  - Heitler, *Quantum Theory of Radiation* (1954): larger \( J_z \) possible if the EM wave is constrained in the transverse plane (cylindrical waves)
  - Spherical waves: expansion in terms of angular momentum eigenfunctions, position dependence of vector potential \( A_\mu(x) \) contains OAM information
  - Beams of light with azimuthal beam dependence \( \exp(\imath l \phi) \) (e.g., Laguerre-Gaussian modes) can carry large values of OAM (*Allen et al, 1992*).

Review: Yao, Padgett, *Advances in Optics and Photonics* 3, 161–204 (2011) and references therein
Orbital vs Spin Angular Momentum \textit{(from Yao’11 review)}

The spin angular momentum (SAM) of light is connected to the polarization of the electric field. Light with linear polarization (left) carries no SAM, whereas right or left circularly polarized light (right) carries a SAM of $\pm h$ per photon.

- Quantization of light beams having azimuthal phase dependence $\exp(i\ell\phi)$ leads to a concept of \textit{twisted photons}
Twisted Photons

- Quantization of light beams having azimuthal phase dependence $\exp(il\phi)$ lead to a concept of twisted photons


The typical transverse intensity pattern of a light beam with orbital angular momentum, (a) theory (b) experiment. The light beam exhibits a dark spot in the center, and a ring-like intensity profile. (c) Azimuthal dependence of beam phase results in a helical wavefront. (d) Orientation of the local momentum of the beam has a vortex pattern (hence another name, *an optical vortex*).
Generation of Light Beams with Orbital Angular Momentum


- Spiral Phase Plates: Gaussian beam is passed through optical media, with azimuthal dependence in thickness
Generation of Twisted Photons with Helical Undulators


- AA, Mikhailichenko, On Generation of Photons Carrying Orbital Angular Momentum in the Helical Undulator, E-print: arXiv 1109.1603
  
  - Considered properties of synchrotron radiation by charged particles passing through a helical undulator. Shown that all harmonics higher than the first one radiated in a helical undulator carry OAM. Large K-factors favor large values of OAM for generated radiation.
Helical Undulators

- AA, Mikhailichenko, On Generation of Photons Carrying Orbital Angular Momentum in the Helical Undulator, E-print: arXiv 1109.1603

*Figure 1.* In a moving frame, the radiation has an electron as a point source moving along the circle with the radius \( r' = \frac{\lambda e K}{\gamma} \). In the Lab frame the cone of radiation is tilted toward the direction of motion (z-axis) by the angle \( 1/\gamma \), so the projector-type radiation is emitted from the off-axis location.
Transfer of Angular Momentum

- From Yao et al (2011): for optical wavelengths, transfer of Orbital AM differs from Spin AM.

- Common misconception: magnetic quantum numbers of the excited state are not affected by OAM
Atomic Excitations with High-L Photons

- Twisted photons may enhance (relatively) atomic transitions with large transfer of angular momentum
Twisted Photon State

- For Bessel beam vector potential and plane-wave expansion we use formalism from Jaregui PRA 70, 033415 (2004) and Jentschura&Serbo, PRL 106, 013001 (2011)

- Use plane-wave expansion

\[
|\kappa m, \gamma, k_z \Lambda\rangle = \int \frac{d^2 k_\perp}{(2\pi)^2} a_{\kappa m, \gamma}(\vec{k}_\perp)|\vec{k}, \Lambda\rangle
\]

\[
= \sqrt{\frac{\kappa}{2\pi}} \int \frac{d\phi_k}{2\pi} (-i)^m \gamma e^{i m \gamma \phi_k} |\vec{k}, \Lambda\rangle
\]

\[
a_{\kappa m, \gamma}(\vec{k}_\perp) = (-i)^m \gamma e^{i m \gamma \phi_k} \sqrt{\frac{2\pi}{\kappa}} \delta(\kappa - |\vec{k}_\perp|).
\]

- Plane wave:

\[
\langle 0 | A^\mu(x) |\vec{k}, \Lambda\rangle = \varepsilon^\mu_{\vec{k}, \Lambda} e^{-ikx}
\]

- Twisted wave:

\[
A'^\mu_{\kappa m, \gamma, k_z \Lambda}(x) = \langle 0 | A^\mu(x) |\kappa m, \gamma, k_z \Lambda\rangle
\]

\[
= \sqrt{\frac{\kappa}{2\pi}} \int \frac{d\phi_k}{2\pi} (-i)^m \gamma e^{i m \gamma \phi_k} \varepsilon^\mu_{\vec{k}, \Lambda} e^{-ikx}
\]
Fields of the Twisted Wave

- Vector potential

\[ A_{\lambda k_m k_2 \lambda}^\mu (x) = e^{-i(\omega t - k_2 z)} \sqrt{\frac{k_2}{2\pi}} \left\{ \frac{\Lambda}{\sqrt{2}} e^{im_\gamma \phi_\rho} \sin \theta_k J_{m_\gamma}(\kappa \rho) \eta_0^\mu 
\]

\[ + i^{-\Lambda} e^{i(m_\gamma - \Lambda) \phi_\rho} \cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(\kappa \rho) \eta_\Lambda^\mu 
\]

\[ + i^{\Lambda} e^{i(m_\gamma + \Lambda) \phi_\rho} \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(\kappa \rho) \eta_-^\mu \right\} .\]

- Poynting vector

\[ S_\rho = 0 , \]

\[ S_\phi = \frac{\kappa \omega^2}{4\pi} \sin \theta_k J_{m_\gamma}(\kappa \rho) 
\]

\[ \times \left( \cos^2 \frac{\theta_k}{2} J_{m_\gamma - 1}(\kappa \rho) + \sin^2 \frac{\theta_k}{2} J_{m_\gamma + 1}(\kappa \rho) \right) , \]

\[ S_z = \frac{\kappa \omega^2}{4\pi} \left( \cos^4 \frac{\theta_k}{2} J_{m_\gamma - 1}(\kappa \rho) - \sin^4 \frac{\theta_k}{2} J_{m_\gamma + 1}(\kappa \rho) \right) \]

Magnetic field

\[ B_\rho = i\omega \sqrt{\frac{\kappa}{4\pi}} e^{i(k_2 z - \omega t + m_\gamma \phi)} 
\]

\[ \times \left( \sin^2 \frac{\theta_k}{2} J_{m_\gamma + 1}(\kappa \rho) + \cos^2 \frac{\theta_k}{2} J_{m_\gamma - 1}(\kappa \rho) \right) \]

\[ B_\phi = \omega \sqrt{\frac{\kappa}{4\pi}} e^{i(k_2 z - \omega t + m_\gamma \phi)} 
\]

\[ \times \left( \sin^2 \frac{\theta_k}{2} J_{m_\gamma + 1}(\kappa \rho) - \cos^2 \frac{\theta_k}{2} J_{m_\gamma - 1}(\kappa \rho) \right) \]

\[ B_z = \omega \sqrt{\frac{\kappa}{4\pi}} e^{i(k_2 z - \omega t + m_\gamma \phi)} \sin^2 \theta_k J_{m_\gamma}(\kappa \rho) , \]
Transverse Beam Profile

. OAM light beam is characterized with a special transverse profile (example from AA, Carlson, Mukherjee,)
. Intensity dip on the beam axis, with transverse size > wavelength
Atomic Photoexcitation

- Interaction Hamiltonian:
  \[ H_1 = -\frac{e}{m_e} \vec{A} \cdot \vec{p}, \]

- Matrix element of the transition:
  \[ S_{fi} = -i \int dt \langle n_f l_f m_f | H_1 | n_i l_i m_i ; \kappa m_\gamma k_z \Lambda \rangle \]

- \( b \)- an impact parameter w.r.t. the atomic center
Matrix Element of Photoexcitation

- Photo-excite a hydrogen atom from the ground to the state with a principal quantum number \( n_f \), OAM \( l_f \), and OAM projection \( m_f \). Incoming twisted photon is defined by AM projection \( m_\gamma \), energy \( \omega \) and a pitch angle \( \theta_k \) (with \( \kappa = k_{\text{perp}} \)).

- Matrix element:

\[
S_{fi} = -2\pi \delta(E_f - E_i - \omega) \frac{e}{m_e a_0} \sqrt{\frac{2\pi\kappa}{3}} e^{i(m_\gamma - m_f)\phi_b} J_{m_f - m_\gamma}(\kappa b)
\times i^{-\Lambda} \left\{ \frac{\cos^2 \frac{\theta_k}{2}}{2} g_{n_f l_f m_f \Lambda} + \frac{i}{\sqrt{2}} \sin \theta_k g_{n_f l_f m_f 0} - \sin^2 \frac{\theta_k}{2} g_{n_f l_f m_f, -\Lambda} \right\}
\]

\[
\overset{\text{def}}{=} 2\pi \delta(E_f - E_i - \omega) M_{n_f l_f m_f \Lambda}(b).
\]

- Atomic factors

\[
g_{n_f l_f m_f \lambda} = -a_0 \int_0^\infty r^2 dr R_{n_f l_f}(r) R'_{10}(r) \int_{-1}^1 d(\cos \theta_r) J_{m_f - \lambda}(\kappa \rho) Y_{l_f m_f}(\theta_r, 0) Y_{1\lambda}(\theta_r, 0) e^{ik_z z}
\]
Calculation Results

- Matrix elements as a function of an impact parameter $b$

**FIG. 2**: Size of the transition amplitude $|M_{n_f l_f m_f \Lambda}^{(m_\gamma=3)}(b)|$ for particular quantum numbers $n_f, l_f, \Lambda$ and several $m_f$. On the upper graph, $m_f = 3$ is the red solid curve, $m_f = 2$ is orange and medium dashed, $m_f = 1$ is gold and long dashed, $m_f = 0$ is green and dot-dashed, $m_f = -1$ is blue and dotted, and transitions to other $m_f$ are quite small and not plotted. Lower graph is for the final state $l_f = 1$; the state $m_f = 1$ is allowed by electric-dipole selection rules for plane waves, while $m_f = 0, -1$ are unique for the twisted photons.
Normalized transition probabilities

- Fermi’s Golden Rule: Squared amplitudes times phase space volume -> excitation rates
- Cross section: transition probability per photon
  - Need to divide squares of above amplitudes by the photon flux
  - Reminder: local photon flux is zero at optical vortex center
    - But we see some amplitudes nonzero at the center (for $\Delta m = m_\gamma$) => singularity!
- Example of photon flux as a function of impact parameter $b$: 
Normalized Transition Probabilities: Electric Dipole

- Surprise or not? Rates for standard electric-dipole transitions scale with the (local) photon flux

\[
\sigma_{1S \rightarrow 2P}(\Lambda=1, m_{\gamma}=1) / \sigma_{1S \rightarrow 2P}(\Lambda=1, m_{\gamma}=3)
\]

Relative probability (per photon) to excite L=1 state
Normalized Transition Probabilities: Higher Multipoles

- Higher multipoles are generated more efficiently for OAM photons near the optical vortex center. Singularity occurs when magnetic quantum number of excited state matches total AM projection of a twisted photon.
- 10x enhancement at $b \sim 0.4$ wavelength!
Helicity Asymmetry

- Flip photon helicity $\Lambda$, keep the OAM projection the same $\Rightarrow$
  - Results a different twisted photon state with $m_\gamma \rightarrow m_\gamma - 2\Lambda$
- Photoabsorption cross sections are different for a given impact parameter $\Rightarrow$
  (parity-conserving) helicity asymmetry; physics reason: flipping helicity affects local photon flux
- The largest asymmetry is near the center of optical vortex
- Asymmetry is zero after averaging over the impact parameter $b$
- May be observed for small-size targets or near-field geometry
Experiments with Atoms

- Confirmed our theoretical arguments
How to Generate Twisted Photons in MeV-GeV?


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**FIG. 2 (color).** Initial (above) and final (below) states for the head-on Compton backscattering geometry of a twisted photon. According to Eq. (24), the conical momentum spread $\kappa$ of the initial twisted photon is preserved ($\kappa' = \kappa$) during the scattering, but the propagation energy increases: $\omega' = \sqrt{k_z^2 + \kappa'^2} \gg \omega = \sqrt{k_z^2 + \kappa^2}$. 
Twisted Photons for Nuclei vs Atoms

- For atomic transitions photon OAM is preferably transferred to internal degrees of freedom if the target is near the center of an optical vortex.
- The nuclear transitions of different multipolarity can be controlled by generating a “twisted” gamma-ray beam.
  - For example, strength of E1 vs E2 transition.
  - Enforce population of high OAM, longer-lived states in nuclei.
    - A step toward a gamma laser?
- Quantum Core area of “efficient” OAM transfer appears large, at a fraction of the twisted-photon wavelength.
Summary

- Considered atomic photoexcitation with twisted photons
  - Results in excitation of states with a range of quantum numbers, different from plane waves
    - Effect is relative suppression of lower-multipole transitions
  - Extending the same approach to photo-nuclear transitions (e.g., E1 vs E2 strengths)
- Plan experiments with existing Compton sources